UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2007 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2007 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	Obtain indefinite integral of the form $a\ln(2x-1)$, where $a = \frac{1}{2}$, 1, or 2	M1	
	Use limits and obtain equation $\frac{1}{2}\ln(2k-1) = 1$	A1	
	Use correct method for solving an equation of the form $a\ln(2k-1) = 1$, where $a = \frac{1}{2}$, 1, or 2, for k	M1	
	Obtain answer $k = \frac{1}{2}(e^2 + 1)$, or exact equivalent	A1	**[4]**
2	*EITHER*: Attempt division by $x^2 + x + 2$ reaching a partial quotient of $x^2 + kx$	M1	
	Complete the division and obtain quotient $x^2 - x + 2$	A1	
	Equate constant remainder to zero and solve for a	M1	
	Obtain answer $a = 4$	A1	
	OR: Calling the unknown factor $x^2 + bx + c$, obtain an equation in b and/or c, or state without working two coefficients with the correct moduli	M1	
	Obtain factor $x^2 - x + 2$	A1	
	Use $a = 2c$ to find a	M1	
	Obtain answer $a = 4$	A1	**[4]**
3	Using 1 and ln x as parts reach $x\ln x \pm \int x.\frac{1}{x}\,dx$	M1*	
	Obtain indefinite integral $x\ln x - x$	A1	
	Substitute correct limits correctly	M1(dep*)	
	Obtain given answer	A1	**[4]**
4	**(i)** Use correct product or quotient rule	M1	
	Obtain derivative in any correct form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = \frac{1}{4}\pi$ or 0.785 with no errors seen	A1	**[4]**
	(ii) Use an appropriate method for determining the nature of a stationary point	M1	
	Show the point is a maximum point with no errors seen	A1	**[2]**
	[SR: for the answer 45° deduct final A1 in part (i), and deduct A1 in part (ii) if this value in degrees is used in the exponential.]		
5	**(i)** Use correct tan($A + B$) formula to obtain an equation in tan x	M1*	
	Use tan 45° = 1	M1(dep*)	
	Obtain the given answer	A1	**[3]**
	(ii) Make reasonable attempt to solve the given quadratic for one value of tan x	M1	
	Obtain $\tan x = -1 \pm \sqrt{2}$, or equivalent in the form $(a \pm \sqrt{b})/c$ (accept 0.4, –2.4)	A1	
	Obtain answer $x = 22.5°$	A1	
	Obtain second answer $x = 112.5$ and no others in the range	A1	**[4]**
	[Ignore answers outside the range.]		
	[Treat answers in radians as a MR and deduct one mark from the marks for the angles.]		

6	**(i)** Make a recognisable sketch of an appropriate graph, e.g. $y = \ln x$	B1	
	Sketch an appropriate second graph, e.g. $y = 2 - x$, correctly and justify the given statement	B1	**[2]**
	(ii) Consider sign of $2 - x - \ln x$ when $x = 1.4$ and $x = 1.7$, or equivalent	M1	
	Complete the argument with correct calculations	A1	**[2]**
	(iii) Rearrange the equation $x = \frac{1}{3}(4 + x - 2\ln x)$ as $2 - x = \ln x$, or *vice versa*	B1	**[1]**
	(iv) Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.56	A1	
	Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.555, 1.565)	A1	**[3]**
7	**(i)** Separate variables correctly and attempt integration of both sides	M1*	
	Obtain term $\ln N$, or equivalent	A1	
	Obtain term $\frac{k}{0.02}\sin(0.02t)$, or equivalent	A1	
	Use $t = 0$, $N = 125$ to evaluate a constant, or as limits, in a solution containing terms of the form $a\ln N$ and $b\sin(0.02t)$, or equivalent	M1	
	Obtain any correct form of solution, e.g. $\ln N = 50k\sin(0.02t) + \ln 125$	A1	**[5]**
	(ii) Substituting $N = 166$ and $t = 30$, evaluate k	M1(dep*)	
	Obtain $k = 0.0100479\ldots$(accept $k = 0.01$)	A1	**[2]**
	(iii) Rearrange and obtain $N = 125\exp(0.502\sin(0.02t))$, or equivalent	B1	
	Set $\sin(0.02t) = -1$ in the expression for N, or equivalent	M1	
	Obtain least value 75.6 (accept answers in the interval [75, 76])	A1	**[3]**
	[For the B1, accept 0.5 following $k = 0.01$, and allow 4.8 or better for ln 125.]		
8	**(a) (i)** *EITHER*: Carry out multiplication of numerator and denominator by $1 + 2i$, or equivalent	M1	
	Obtain answer $2 + i$, or any equivalent of the form $(a + ib)/c$	A1	
	*OR*1: Obtain two equations in x and y, and solve for x or for y	M1	
	Obtain answer $2 + i$, or equivalent	A1	
	*OR*2: Using the correct processes express z in polar form	M1	
	Obtain answer $2 + i$, or equivalent	A1	**[2]**
	(ii) State that the modulus of z is $\sqrt{5}$ or 2.24	B1	
	State that the argument of z is 0.464 or 26.6°	B1	**[2]**
	(b) *EITHER*: Square $x + iy$ and equate real and imaginary parts to 5 and -12 respectively	M1	
	Obtain $x^2 - y^2 = 5$ and $2xy = -12$	A1	
	Eliminate one variable and obtain an equation in the other	M1	
	Obtain $x^4 - 5x^2 - 36 = 0$ or $y^4 + 5y^2 - 36 = 0$, or 3-term equivalent	A1	
	Obtain answer $3 - 2i$	A1	
	Obtain second answer $-3 + 2i$ and no others	A1	
	[SR: Allow a solution with $2xy = 12$ to earn the second A1 and thus a maximum of 3/6.]		
	OR: Convert $5 - 12i$ to polar form (R, θ)	M1	
	Use the fact that a square root has the polar form $(\sqrt{R}, \frac{1}{2}\theta)$	M1	
	Obtain one root in polar form, e.g. $(\sqrt{13}, -0.588)$ or $(\sqrt{13}, -33.7°)$	A1 + A1	
	Obtain answer $3 - 2i$	A1	
	Obtain answer $-3 + 2i$ and no others	A1	**[6]**

9 **(i)** State or imply the form $\frac{A}{1-x}+\frac{B}{1+2x}+\frac{C}{2+x}$ B1

Use any relevant method to determine a constant M1

Obtain $A = 1$, $B = 2$ and $C = -4$ A1 + A1 + A1 **[5]**

(ii) Use correct method to obtain the first two terms of the expansion of $(1-x)^{-1}$, $(1+2x)^{-1}$, $(2+x)^{-1}$, or $(1+\frac{1}{2}x)^{-1}$ M1

Obtain complete unsimplified expansions up to x^2 of each partial fraction A1√ + A1√ + A1√

Combine expansions and obtain answer $1-2x+\frac{17}{2}x^2$ A1 **[5]**

[Binomial coefficients such as $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient for the M1. The f.t. is on A, B, C.]

[Apply this scheme to attempts to expand $(2-x+8x^2)(1-x)^{-1}(1+2x)^{-1}(2+x)^{-1}$, giving M1A1A1A1 for the expansions, and A1 for the final answer.]

[Allow Maclaurin, giving M1A1√A1√ for f(0) = 1 and f ′(0) = –2, A1√ for f ″(0) = 17 and A1 for the final answer (f.t. is on A, B,C).]

10 **(i)** Substitute for **r** and expand the given scalar product, or correct equivalent, to obtain an equation in s M1

Solve a linear equation formed from a scalar product for s M1

Obtain $s = 2$ and position vector 3**i** + 2**j** + **k** for A A1 **[3]**

(ii) State or imply a normal vector of p is 2**i** –3**j** + 6**k**, or equivalent B1

Use the correct process for evaluating a relevant scalar product, e.g. (**i** – 2**j** + 2**k**).(2**i** –3**j** + 6**k**) M1

Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result M1

Obtain final answer 72.2° or 1.26 radians A1 **[4]**

(iii) *EITHER*: Taking the direction vector of the line to be a**i** + b**j** + c**k**, state equation $2a - 3b + 6c = 0$ B1

State equation $a - 2b + 2c = 0$ B1

Solve to find one ratio, e.g. $a : b$ M1

Obtain ratio $a : b : c = 6 : 2 : -1$, or equivalent A1

State answer **r** = 3**i** + 2**j** + **k** + λ(6**i** + 2**j** –**k**), or equivalent A1√

*OR*1: Attempt to calculate the vector product of a direction vector for the line l and a normal vector of the plane p, e.g.(**i** –2**j** + 2**k**)×(2**i** –3**j** + 6**k**) M2

Obtain two correct components of the product A1

Obtain answer –6**i** –2**j** + **k**, or equivalent A1

State answer **r** = 3**i** + 2**j** + **k** + λ(–6**i** –2**j** + **k**), or equivalent A1√

*OR*2: Obtain the equation of the plane containing A and perpendicular to the line l M1

State answer $x - 2y + 2z = 1$, or equivalent A1√

Find position vector of a second point B on the line of intersection of this plane with the plane p, e.g. 9**i** + 4**j** M1

Obtain a direction vector for this line of intersection, e.g. 6**i** + 2**j** – **k** A1

State answer **r** = 3**i** + 2**j** + **k** + λ(6**i** + 2**j** – **k**), or equivalent A1 **[5]**

[The f.t. is on A.]